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Neutron disappearance inside the nucleus

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Abstract

We consider the possibility that a neutron may disappear inside the nucleus, which will demonstrate the existence of baryon violating $\Delta B = 1$ interactions. It has recently been proposed that such a process may have an effect on the free neutron decay life time. We evaluate the widths for $n \rightarrow \chi$ and $n \rightarrow \chi\gamma$, with χ being a light dark matter particle emitted by a loosely bound neutron in various light nuclei. We find that, assuming a mass m_χ close to 938 MeV, the obtained width for $n \rightarrow \chi$ in ^{11}Be is much larger than the observed beta decay width. This suggests a severe limit on the possible decay channel of $n \rightarrow \chi\gamma$ for free neutron.

Keywords: neutron decay, neutron lifetime, halo nuclei, baryon number violation

(Some figures may appear in colour only in the online journal)

1. Introduction

The neutron is one of the building blocks of matter. Without it, complex atomic nuclei simply would not have formed. Although the neutron was discovered over eighty years ago and has been studied intensively thereafter, its precise lifetime is still an open question [1, 2]. There are two qualitatively different types of direct neutron lifetime measurements: bottle and beam experiments. In the first method one obtains [3]:

$$t_n(\text{bottle}) = (879.6 \pm 0.6)\,\text{s}. \tag{1}$$

In the second, the beam method, the result as given by Particle Data Group average [4, 5] is

$$\tau_n(\text{beam}) = (888 \pm 2.0)\,\text{s}. \tag{2}$$

The discrepancy between the two results is 4.0σ.

This suggests that either one of the measurement methods suffers from an uncontrolled systematic error, or there is a physics reason of why the two methods give different results, involving very interesting physics. It is interesting to note that in the beam experiment the life time is longer.

The above facts were known to the authors of a recent paper [6] and, in particular, the problem of the discrepancy between the two experimental measurements of the neutron decay lifetime has been addressed in their work. They noted that, since in the 'beam' experiment the result is obtained by studying the decay, the lifetime they measure is related to the actual neutron lifetime by

$$\tau_n(\text{beam}) = \frac{\tau_n}{\text{Br}(n \rightarrow p + \text{anything})}. \tag{3}$$

These authors suggest that the discrepancy can be explained by considering an extra channel in the beam experiment, which involves the emission of a dark fermion particle χ, which goes undetected. Then they proposed a model which can give a branching ratio of 1% to this new channel, while the standard channel covers only 99%, thus settling the issue. It is, however, necessary in their treatment to assume that this new particle is a neutral Dirac like fermion with a mass slightly lighter than that of the neutron. If it were a Majorana like particle this mechanism could lead to neutron-antineutron oscillations, thus being in conflict with the non observation of such oscillations.

For a free neutron this cannot occur without the emission of another particle, e.g. a photon to conserve energy momentum, which at the same time provides an interesting experimental signature for the suggested mechanism.

Since the emitted particle is assumed not to carry any baryon number [6], this scenario is very interesting, since if true, it will demonstrate the existence of baryon number violating $\Delta B = 1$ interactions. This scenario seems, however, to be excluded from astrophysical data involving neutron stars [7–9]. Such arguments, however, do not apply, if there exists a Coulomb repulsion between the dark fermions mediated by a dark photon [10]. In any case the neutron star arguments apply only to neutrons bound by gravity and not by strong interactions.

The neutron in the nucleus seems to behave differently, due to the nuclear binding. In certain cases it decays like in the β decay, but the produced proton cannot escape due to nuclear binding, while a daughter nucleus appears with its charge increased by one unit. Decays of well-bound nucleons (neutrons) into invisible particles have been searched by measuring γ rays long time ago [11, 12]. In the model considered above the produced dark matter particle χ, interacting very weakly, can escape. In this case energy-momentum can be conserved without the emission of additional particles, like the photon, and the decay width is expected to be much larger.

In this work we will consider the possibility that such a process may occur inside the nucleus:

$$A(N, Z) \rightarrow A(N-1, Z)^* + \chi, \tag{4}$$

where χ is the dark matter fermion. Since χ is also supposed to be produced in the decay of the free neutron, it must be lighter than the neutron. In fact to explain the neutron life discrepancy it was necessary to assume [6] its mass must be less than but very close to that of the neutron $m_n = 939.565$:

$$937.900 \text{ MeV} < m_\chi < 938.783 \text{ MeV}, \tag{5}$$

where m_χ is the dark particle mass. The lower and upper bounds come from the stability of ^{9}Be and the fact that the decay $\chi \rightarrow p + e^- + \bar{\nu}_e$ is forbidden, respectively. Note that the lower bound corresponds to $m_n - B_n$ with $B_n = 1.67$ MeV being the neutron binding energy of ^{9}Be.

While our paper was in preparation a related paper appeared [13], but it was based on experimental information. In any case we cannot see how the factor g_A they employ could appear in a particle model like ours, with the baryon violating interaction mediated by scalar particles.

Before proceeding to a further study of the baryon number violating process, we will consider some facts about the nuclei, which can serve as targets: Nuclei that need to be studied experimentally should have a loosely bound neutron with $B_n \approx 0.5$ MeV–1 MeV to decay to χ, as in equations (4) and (5), and a long β decay half-life of the order of seconds in order to make the possible decay with much shorter half-life well separated from the β decay. The long half-life is necessary to make the decay to χ visible. This way we have two possible candidates, ^{11}Be and ^{15}C. Thus, we will mainly discuss the possible decay of a bound neutron to the ground state in the residual nucleus in these two cases.

^{11}Be has already been studied experimentally, see, e.g., [14] and references there in. Its ground state is $1/2^+$ and the first excited state is $1/2^-$. It seems that the $1/2^+$ is an unusual ordering from the point of view of the simple shell model. It has also been studied both in the context of effective field theory [15], focusing on its electric properties, and the no-core–shell model with continuum (NCSMC) type calculations [16]. The later an *ab initio* calculation, has succeeded in getting the inversion right. Thus the $1/2^+$ is composed of a major component of the form ^{10}Be g.s $\otimes\ 1s_{1/2}(n)$ and, possibly, of another one of the type $^{10}\mathrm{Be}2^+ \otimes 0d_{5/2}(n)$. The first can decay into the ground state (g.s.) of ^{10}Be.

On the theoretical side there have been a lot of additional studies [17–19]. Variational Model approach [20] as well as models which vary the single energies via vibrational and rotational core couplings, succeed in reproducing the needed level inversion in a rather systematic manner. One can say that a common feature for the success of these models is the inclusion of core excitation. On the other hand *ab initio* No-Core Shell Model calculations [21] have not been able to reproduce this level inversion. Anyway it is considered a success that a significant drop in the energy of the $1/2^+$ state in ^{11}Be is reported with enlarging the model space.

Relevant useful experimental as well as theoretical information can be found in a previous work [22], in particular the fact that the relevant spectroscopic factor for the first component is about $(80 \pm 10)\%$.

2. The formalism

2.1. Neutron bound wave functions

We will consider the neutron as a bound state of three quarks in a color singlet s-state. The orbital part of the form:

$$\Psi(R,\ \xi,\ \eta) = \Phi(R)\psi_{0s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}). \tag{6}$$

The $\psi_{0s}(\boldsymbol{\xi})$ and $\psi_{0s}(\boldsymbol{\eta})$ are bound wave functions dependent on the relative internal variables, which, for simplicity, we will assume to be of the $0s$ harmonic oscillator type, so that one can easily separate out the internal coordinates. Thus

$$\psi_{0s}(\xi) = \sqrt{\frac{1}{\pi\sqrt{\pi}}}\frac{1}{(b_N)^{3/2}}\mathrm{e}^{-\frac{\xi^2}{2b_N^2}},\ \ \xi = \frac{1}{\sqrt{2}}(\mathbf{x}_1 - \mathbf{x}_2). \tag{7}$$

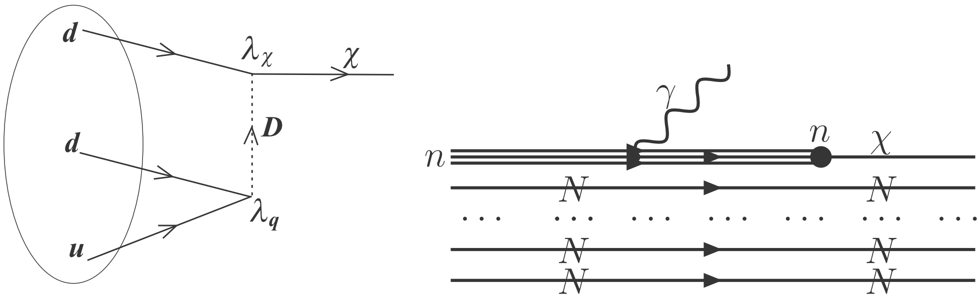

Figure 1. Left panel: a dark matter colorless Dirac fermion is emitted from a neutron (n) viewed as a bound state of three quarks. At the quark level the baryon number violating process is mediated by a colored scalar field D. Right panel: a bound neutron in a nucleus is converted into a dark matter particle. Before this happens it may also emit a photon, which in the nucleus is not necessary. The other $A - 1$ nucleons (N) do not participate in the process.

Similarly

$$\psi_{0s}(\eta) = \sqrt{\frac{1}{\pi\sqrt{\pi}}}\frac{1}{(b_N)^{3/2}}\mathrm{e}^{-\frac{\eta^2}{2b_N^2}}, \;\; \eta = \frac{1}{\sqrt{6}}(\mathbf{x}_1 + \mathbf{x}_2 - 2\mathbf{x}_3). \tag{8}$$

The center of mass coordinate is taken to be:

$$\mathbf{R} = \frac{1}{\sqrt{3}}(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3) \tag{9}$$

with $\mathbf{x}_i$, $i = 1, 2, 3$ the quark coordinates and b_N the nucleon size parameter related to the nucleon radius R_N via the relation $R_N^2 = (3/2)b_N^2$. The functions $\phi(\boldsymbol{\xi})$ and $\phi(\boldsymbol{\eta})$ are normalized in the usual way.

2.2. The amplitude for neutron decay to a dark mater fermion

The process derived from the model of [6] is exhibited in figure 1. The orbital part of the amplitude associated with this process takes the form:

$$\mathcal{M} = \frac{\lambda_q\lambda_\chi}{m_\Phi^2}\psi_{0s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta})(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{q}), \tag{10}$$

where λ_q and λ_χ are the Yukawa couplings of the scalar field D as shown in figure 1, with m_Φ the mass of the scalar. $\mathbf{p}_i$, $i = 1, 2, 3$ are the quark momenta and $\mathbf{q}$ the momentum of the outgoing momentum of the dark particle.

The Fourier transform of the amplitude in coordinate space becomes:

$$\mathcal{M} = \frac{1}{(2\pi)^6}\frac{\lambda_q\lambda_\chi}{m_\Phi^2}\psi_{0s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta})\int \mathrm{d}^3\mathbf{p}_1\int \mathrm{d}^3\mathbf{p}_2 \times \int \mathrm{d}^3\mathbf{p}_3 \mathrm{e}^{\mathrm{i}\mathbf{p}_1\cdot\mathbf{x}_1}\mathrm{e}^{\mathrm{i}\mathbf{p}_2\cdot\mathbf{x}_2}\mathrm{e}^{\mathrm{i}\mathbf{p}_3\cdot\mathbf{x}_3}\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{q}) \tag{11}$$

or

$$\mathcal{M} = \frac{\lambda_q\lambda_\chi}{m_\Phi^2}\psi_{0s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta})\delta(\mathbf{x}_1 - \mathbf{x}_3)\delta(\mathbf{x}_2 - \mathbf{x}_3)\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{x}_3} \tag{12}$$

or

$$\mathscr{M} = \frac{\lambda_q \lambda_\chi}{m_\Phi^2} \psi_{0s}(\boldsymbol{\xi}) \psi_{0s}(\boldsymbol{\eta}) \delta(\sqrt{2}\boldsymbol{\xi}) \delta\left(\sqrt{\frac{3}{2}}\boldsymbol{\eta}\right) \mathrm{e}^{\mathrm{i}\frac{\mathbf{q}}{\sqrt{3}}.(-\sqrt{2}\boldsymbol{\eta}+\mathbf{R})}. \tag{13}$$

Thus we get:

$$\mathscr{M} = \frac{1}{2\sqrt{2}} \frac{2\sqrt{2}}{3\sqrt{3}} \frac{\lambda_q \lambda_\chi}{m_\Phi^2} |\psi(0)|^2 \mathrm{e}^{\mathrm{i}\frac{\mathbf{q}}{\sqrt{3}}.\mathbf{R}}. \tag{14}$$

2.3. The spin-isospin-color factor

We must now consider the internal degrees, i.e. spin-isospin-color factor. We will employ group theoretical techniques. Indeed the neutron isospin $I = 1/2$ implies that the corresponding $SU(2)$ symmetry corresponds to a Young tableaux $[f^I] = [2, 1]$. Since the orbital part for $0s$ quarks is completely symmetric, the spin color part for a completely antisymmetric neutron wave function must correspond to the associated Young tableaux, i.e. that obtained from the previous by interchanging rows and columns, which in this case is $[f^{sc}] = [2, 1]$ again, which, of course, contains [23] a color singlet with spin $1/2$, $[2, 1](0, 0)1/2$. The two quarks appearing in the process of figure 1, symbolically indicated a ud, must be in a spin zero state, in order to couple to a scalar particle, and must be in a color anti-triplet. Thus they must be in an isospin zero state, which means that the isospin coefficient of fractional parentage (CFP) is unity. The spin-color one particle CFP connecting one quark to the two quark color anti-triplet, i.e. $[2](0, 1)s = 0$, is also unity [23]. So the corresponding matrix element is $\sqrt{3}\sqrt{\frac{\dim[2]}{\dim[2,1]}} = \sqrt{\frac{3}{2}}$, where the first number corresponds to the number of quarks in the neutron and the second comes from the dimensions of the relevant representations of the symmetric group. Thus finally

$$\mathscr{M} = \kappa_{\text{scale}} \mathrm{e}^{\mathrm{i}\frac{\mathbf{q}}{\sqrt{3}}.\mathbf{R}}, \quad \kappa_{\text{scale}} = \frac{1}{3\sqrt{2}}\epsilon, \; \epsilon = |\psi(0)|^2 \frac{\lambda_q \lambda_\chi}{m_\Phi^2}, \tag{15}$$

where

$$|\psi(0)|^2 = \frac{1}{\pi\sqrt{\pi}} \frac{1}{b_N^3}, \quad b_N = \sqrt{\frac{2}{3}} R_N \tag{16}$$

is the baryon density at the origin.

2.4. Fixing the nucleon size parameter

For a typical value of $R_N = 0.8$ fm we obtain a value of $|\psi(0)|^2 = 0.005\,\text{GeV}^3$. This is a bit smaller than the analogous parameter β employed in the free neutron decay [6], i.e. $\beta = 0.014\,\text{GeV}^3$, obtained recently from lattice computation of proton decay matrix elements [24]. We will adopt the value of $b_N = 0.5$ fm to be consistent with the larger value of $\beta = 0.014\,\text{GeV}^3$. Thus

$$\kappa_{\text{scale}} = \frac{1}{3\sqrt{2}} \frac{1}{\pi\sqrt{\pi}} \frac{1}{b_N} \frac{\lambda_q \lambda_\chi}{(b_N m_\Phi)^2}. \tag{17}$$

We find it convenient to indicate the size of the baryon number violating neutron conversion to a dark mater fermion by the dimensionless quantity $s = \left(\frac{\lambda_q \lambda_\chi}{(b_N m_\Phi)^2}\right)^2$ instead of the dimensionful parameter ϵ^2 used in [6]. Using the value of $(\lambda_q \lambda_\chi)/m_\Phi^2 = 6.7 \times 10^{-6}\,\text{TeV}^{-2}$, i.e. the one employed in the free nucleon exotic decay to dark matter [6], we find that both are indeed very small:

$$s \approx 1.5 \times 10^{-24} \text{ or } \epsilon^2 = 8.8 \times 10^{-27}\,\text{GeV}^2.$$

Either one can be used in bound as well as in free nucleon decay.

3. Expression for the decay width

The decay width is given by the expression:

$$d\Gamma = \frac{1}{(2\pi)^2} d^3\mathbf{q} d^3\mathbf{P}_A \delta(\mathbf{q} + \mathbf{P}_A)\delta(\Delta - E_x + m_n - m_\chi - T)|\langle \text{ME} \rangle|^2 \quad (18)$$

where $\mathbf{P}_A$ and $\mathbf{q}$ are the momenta of the final nucleus and the outgoing dark matter particle χ, respectively, with the latter's mass being m_χ and its kinetic energy T. Δ is the difference of the ground state energies of the nuclei involved with the neutron mass separated out and E_x the excitation energy of the populated final nuclear state. Finally ME (matrix element) is the invariant amplitude which will be been given in in equation (36) of the appendix. Thus

$$\Gamma = \frac{1}{\pi}\sqrt{2m_\chi(\Delta - E_x + m_n - m_\chi)}\, m_\chi |\langle \text{ME} \rangle|^2$$
$$\approx \frac{1}{\pi}\sqrt{2m_n(\Delta - E_x + m_n - m_\chi)}\, m_n |\langle \text{ME} \rangle|^2 \quad (19)$$

or, indicating by $j(n)$ the angular momentum of the loosely bound neutron, we obtain:

$$\Gamma = \frac{16}{9\pi^3}\frac{a_N^3}{b_N^2}\left(\frac{\lambda_q\lambda_\chi}{(b_N m_\Phi)^2}\right)^2 \sqrt{2m_n(\Delta - E_x + m_n - m_\chi)}$$
$$\times\, m_n \langle A(N-1, Z)J_f; j(n); A(N, Z)\rangle^2 2 f_{j,\ell}^2 (F_{n_r\ell}(\alpha))^2, \quad (20)$$

with $F_{n_r\ell}(\alpha)$ the relevant form factor defined by equation (31) and α given by equation (23) below.

The function $f_{j,\ell}^2$ is an angular momentum dimension coefficient, which in our case is trivial, i.e. $f_{j,\ell}^2 = 1/2$, see equation (30). The form factors for harmonic oscillator wave functions have been obtained analytically in the appendix. We will, however, illustrate their behavior in figure 2. The needed nuclear CFP's can be calculated in a shell model treatment or perhaps they can be extracted from other experiments as mentioned earlier.

We note that in the interesting case for $1s$ orbital, the form factor is sensitive to the nuclear model employed, since this form factor essentially involves the overlap of a bound nucleon wave function with a plane wave. The result is particularly sensitive to the parameters for nucleon wave functions that have nodes, like the $1s$ orbit, since at some point there appears a change in sign. Thus, due to the r^2 factor appearing in the integral, the behavior at large r becomes crucial. One might have thought that large additional suppression may be due to the oscillations of the spherical Bessel function $j_0(q\, r)$, see equation (28). In the kinematic region of interest to us, this is not the case, since the spherical Bessel function does not make many oscillations, see figure 2. It merely changes sign at around 16 fm, but all nuclear wave functions are small there.

In view of the above, the harmonic oscillator description, however, may not be satisfactory in describing the conversion of a nucleon in the nucleus into a dark matter particle,

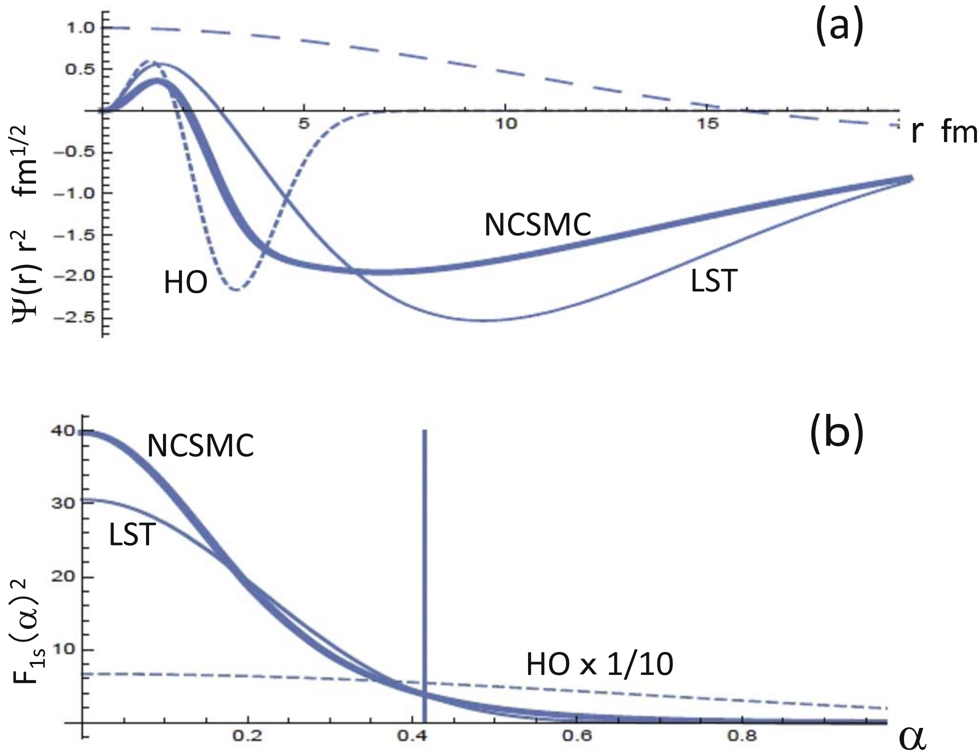

Figure 2. In the top panel (a) we plot the expression $r^2\Psi(r)$, with $\Psi(r)$ the wave function of the loosely bound neutron in ^{11}Be as follows: solid line corresponds to the empirical local scale transformations (LST), thick solid line to the NCSMC and the dashed line to the harmonic oscillator case. One clearly sees the importance of the sign change in the nuclear wave function. We also exhibit the behavior of the spherical Bessel function $j_0(q\ r)$, which is, of course, dimensionless, for a small momentum transfer of interest to us, i.e. $q = 0.2\ \mathrm{fm}^{-1} \Leftrightarrow m_n - B - m_\chi \approx 0.8$ MeV, (long dashed curve), merely to indicate that it does not make real oscillations. It eventually changes sign, but all nuclear wave functions are small there. In the bottom panel (b) we exhibit the square of the form factors associated with the 1s state of ^{11}Be for the HO oscillator case (dashed line), the LST wave function (solid line) and the NCSMC (thick solid line) are exhibited. Note that the presented here HO results have been multiplied by a factor of 10, to make the dashed line visible. The vertical line corresponds to value of α associated with the energy $m_n - B - m_\chi \approx 0.8$ MeV, which was used in the present calculation.

since a neutron with a binding energy of about 1 MeV may extend further than that prescribed by the harmonic oscillator (HO) model.

One, therefore, has to modify these wave functions. To this end we recall that wave functions obtained in the context of the NCSMC [16] as well as in the approach of local scale transformations (LST) [25], have been found useful in the study of extended matter distributions in nuclei. So we are going to employ both of them in the present calculation. In the first approach using the NCSMC wave functions provided to us by Navratil [16]. In the second approach, using the techniques of the LST of Karataglidis and Amos [25], we

Table 1. The form factors squared considered in this work. The first two values of α correspond to ^{11}Be and the last two correspond to ^{15}C.

	^{11}Be	^{11}Be	^{15}C	^{15}C
$m_n - m_\chi - B$	0.8 MeV	1.0 MeV	0.092 MeV	0.115 MeV
α	0.415	0.464	0.140	0.158
$F_{1s}^2(\alpha)$(HO)	0.541	0.514	0.649	0.646
$F_{1s}^2(\alpha)$(NCSMC)	3.82	2.60	—	—
$F_{1s}^2(\alpha)$(LST)	3.79	2.08	14.1	13.5

obtained the needed wave functions as discussed in appendix B. The main relevant results are presented in figure 2.

It should be mentioned that the the harmonic oscillator size parameter employed has been obtained in the usual way:

$$a_N = \frac{\hbar c}{\sqrt{\hbar\omega m_n c^2}},\ \hbar\omega = 41A^{-1/3}\ \text{MeV}. \tag{21}$$

This leads to the value of $a_N = 1.5$ fm for the $A = 11$ nucleus and 1.55 for the $A = 15$ system.

The LST model is not sensitive to this parameter, since the long range behavior of the wave function is of exponential form with a range related to a neutron energy [25] as discussed in the appendix. For short distances we used the above a_N with the transition occurring at the same distance a_N.

In the NCSMC model a somewhat smaller size parameter, 1.44 fm, was employed by Navratil [16], but the long range behavior of his wave function is not sensitive to this choice.

We find that, for a binding energy of 0.51 MeV and the choice of $m = 4$, see appendix B, the LST calculations for 1s wave functions yield an enhancement of the form factor $(F_{n_r\ell}(\alpha))^2$ by about 7.0 times, compared to those obtained with harmonic oscillator wave functions, for a value of α around 0.4 (for the definition of α see equation (23) below). For $m = 2$ the results change slightly, this increases to 7.6. So we will adopt here the value of $m = 4$. In the case of the NCSMC for ^{11}Be we get an enhancement of the form actor of about 7.1 compared to that obtained with the HO. Thus for ^{11}Be the LST and NCSMC are in agreement. The reason that the enhancement is not very large is the fact the parameter α, proportional to the momentum transfer, is quite large (see table 1), for $\delta E = m_n - B - m_\chi = 0.8$. The HO form factor does not fall with α as fast as the other two do.

We now come to the second candidate, namely ^{15}C. Here the spectroscopic factor is determined experimentally to be (0.75 ± 0.15)% and we will adopt the value 0.75 [26] for the 1s neutron. We will use $a_N = 1.55$ both for the HO and the LST calculations. The latter is not sensitive to this value, since, as we have mentioned, the long range behavior of the wave function is dominated by the the neutron binding energy, which is found in standard tables [27] $B = 1.218$ MeV. Because of this larger binding we get $\delta E = m_n - B - m_\chi = 0.092$. The width is enhanced by a factor of 21 compared to that of the HO. The reason that this enhancement is now large is the fact the value of α is small (see table 1). We are not sure of whether a similar enhancement exists in the case of the NCSMC, so we prefer to leave blank the appropriate place in table 2.

Table 2. The expected widths for neutron decay into a dark matter particle χ inside a nucleus for the indicated values of $\delta E = m_n - B - m_\chi$. S stands for stable isotope. We present results obtained with the HO shell model wave function as well as for the NCSMC and LST models discussed in the text. The last two models may be more appropriate for halo type nuclei.

Nucleus	B_n [MeV]	J^π	j_n	$T_{1/2}$[s]	δE [MeV]	$g(\alpha)$ (HO)	$\Gamma[10^{-16}$ eV] (HO)	$g(\alpha)$ (NCSMC)	$\Gamma[10^{-16}$ eV] (NCSMC)	$g(\alpha)$ (LST)	$\Gamma[10^{-16}$ eV] (LST)
^{2}H	2.225	1^+	$0s_{1/2}$	S	—	—	—	—	—	—	—
^{9}Be	1.665	$(3/2)^-$	$0p_{3/2}$	S	—	—	—	—	—	—	—
^{11}Be	0.504	$(1/2)^+$	$1s_{1/2}$	13.8	0.80	0.221	0.20	1.63	1.4	1.61	1.4
^{15}C	1.218	$(1/2)^+$	$1s_{1/2}$	2.45	0.092	0.243	0.073	—	—	5.1	1.6

4. Some results

To simplify matters let us for the moment assume the same nuclear size parameter, e.g. $a_N = 1.5$ fm, for the nuclei considered here and a nucleon size parameter of $b_N = 0.5$ fm. Let us also take the value of $(\lambda_q \lambda_\chi)/m_\Phi^2 = 6.7 \times 10^{-6}\text{TeV}^{-2}$, i.e. the one employed in the sister free nucleon exotic decay to dark matter [6]. In the special case of halo nuclei considered here $\Delta = -\text{B}$ with $B > 0$ the binding energy of the decaying neutron and transitions to the ground state, $E_x = 0$, equation (20) becomes:

$$\Gamma = 1.0 \times 10^{-16}\text{eV}\sqrt{\frac{m_n - B - m_\chi}{1\,\text{MeV}}}\, g(\alpha),$$
$$g(\alpha) = \langle A(N-1, Z)J_f; j(n); A(N, Z)\rangle^2 f_{j,\ell}^2 (F_{n,\ell}(\alpha))^2 \quad (22)$$

with $j(n)$ indicating the state of the loosely bound neutron and

$$\alpha = \sqrt{2}\,\frac{\sqrt{2m_n(m_n - B - m_\chi)}\, a_N}{\hbar c}. \quad (23)$$

In other words the dimensionless parameter α is proportional to the momentum $\sqrt{2m_n(m_n - B - m_\chi)}$ of the emitted dark matter particle χ.

Let us consider the invisible decay of a loosely-bound neutron in a nucleus. It turns out there exist several nuclei to be considered as given in table 2. The deuteron and ^{9}Be are stable isotopes with the half-life much longer than that of the Universe. Then their small limits on the widths exclude the n decays to χ with $m_\chi \leqslant 937.3$ MeV and $m_\chi \leqslant 937.9$ MeV, respectively [6]. Then the χ mass region is limited in a narrow region of 937.9 MeV $\leqslant m_\chi \leqslant$ 939.56 MeV.

We are now in position to estimate the widths in the case of the nuclei of interest. The relevant nuclear parameters are contained in the quantity $g(\alpha)$ and are presented in table 2. Regarding the nuclear input in the case of ^{11}Be we used an average spectroscopic factor $\langle A(N-1,)J_f; j(n); A(N, Z)\rangle^2 = 0.85$ [22] for a transition to the ground state. Also in the case of the 1s HO, NCSMC and LST wave function for various values of $m_n - m_\chi - B$ we find the form factors given in table 1.

The decay half-life of ^{11}Be is measured to be 13.81 ± 0.08 s by counting the decay particle as a function of the time [28]. The width is 3.3×10^{-17}eV, which is much smaller than the evaluated width given by (22), as shown in figure 3. Note that the observed ^{11}Be width is smaller by a factor of about $10 - 3$ compared to the evaluated widths based of the realistic LST and NCSMC form factors in the region of $m_n - m_\chi = 0.8 - 1.6$ MeV.

Thus the decay to the DM with $m_\chi \leqslant 939.0$ MeV is excluded. Note that this limit is very insensitive to the nuclear structure coefficient g given in equation (22). The lighter χ with $m_\chi \leqslant 937.9$ MeV is excluded by the long-lived ^{9}Be [6]. On the other hand the heavier DM with $m_\chi \geqslant 938.78$MeV is excluded since the decay of $\chi \rightarrow p + e^- + \bar{\nu}_e$ is forbidden [6].

Thus the decay of the bound neutron is excluded, and thus the suggested decay of the free neutron to $\chi + \gamma$ is not likely. Note that χ is assumed not to decay to $p + e^- + \bar{\nu}_e$ [6] and thus, if we allow unstable χ, the mass region of 939.565-939.0 is not excluded.

The measured width of 1.9 10^{-16} eV for ^{15}C is of the same order of magnitude as the evaluated ones and, thus, disfavors the possibility of DM with $m_\chi \leqslant 938.3$ MeV. Recently n decays in various nuclei have been discussed [29].

As we have mentioned the radiation of a photon by the bound decaying neutron is not needed. We have , however, estimated in the appendix the branching ratio for such a process. For the most interesting case of $1s_{1/2}$ neutron we obtain:

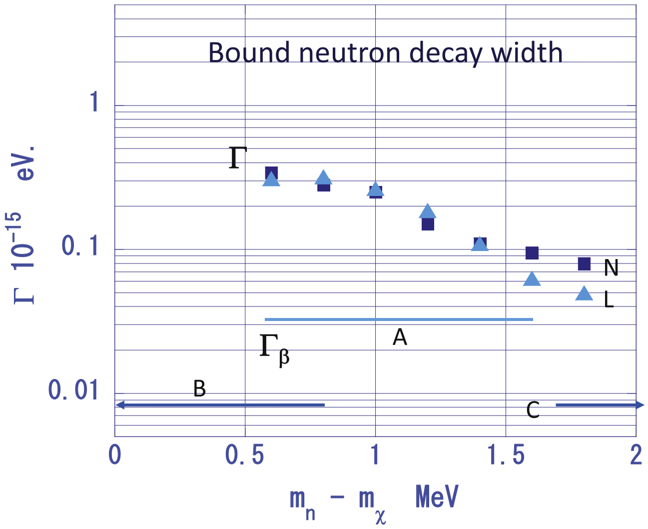

Figure 3. The experimental decay width Γ_β for ^{11}Be (thick line), the evaluated via equation (22) width Γ_χ (squares for model NCSMC, marked as N, and triangles for LST, marked as L), and the excluded mass-regions with A: by the ^{11}Be decay width, B: by the decay to proton, and C: by the ^{9}Be, respectively. The size of the evaluated value reflects the effect of the uncertainty of 15% in the spectroscopic factor. The uncertainty of around 1% in the experimental error in the half-life is within the thickness of the line.

$$\frac{\Gamma(J_i \to J_f\chi\gamma)}{\Gamma(J_i \to J_f\chi)} \approx 2.4 \times 10^{-7}\frac{(m_n - B - m_\chi)^2}{1\text{ MeV}^2}. \tag{24}$$

The width for the radiative free neutron decay is given [6] by:

$$\Gamma(n \to \gamma\chi) = \frac{g_s^2}{8\pi}4\pi\alpha\left(1 - \frac{m_\chi^2}{m_n^2}\right)^3\frac{\epsilon^2}{(m_n - m_\chi)^2}m_n \tag{25}$$

where $\epsilon = 9.38 \times 10^{-14}$ GeV and g_s is the neutron g-factor, $g_s = -3.80$. Here α is the fine structure constant, not to be confused with that of equation (23). Then one obtains:

$$\Gamma(n \to \gamma\chi) \approx \frac{g_s^2}{8\pi}4\pi\alpha\frac{8\epsilon^2}{m_n^2}(m_n - m_\chi) \approx 4 \times 10^{-27}(m_n - m_\chi), \tag{26}$$

which almost the same with the $7 \times 10^{-27}(m_n - m_\chi)$ estimated in [6] to fit the free neutron life time. We thus see that the radiative width for a bound neutron is of the same order with that estimated for a free neutron.

5. Discussion and remarks

Our estimates of the expected widths for baryon number violating neutron decay to a dark matter particle inside the nucleus are summarized in table 2 and figure 3. In figure 3 we have included the results obtained in the context of NCSMC, resulting from an almost *ab initio* constructed wave function, as well as those obtained in the context of the LST. It is encouraging that these two treatments yield results, which essentially agree with each other, and both significantly enhance the widths obtained in the context of the HO shell model.

For a better understanding of figure 3 we note that, there appear two terms in the expression of the width, equation (22). The first is kinematical in nature and increases with the $\sqrt{m_n - B - m_\chi}$. The other comes from the form factor through the α dependence and as a result it decreases with $m_n - B - m_\chi$. The two terms tend in the opposite direction. In the region of interest to us the form factors in the case of NCSMC and LST models decrease very fast as a function of α, see figure 2, and, as a result, for a given B the width tends to decrease with $m_n - m_\chi$. In the case of the HO model the form factor decreases much slower with α and the net result is a width, which slightly increases with $m_n - m_\chi$. Note that the HO wave function does not extend far enough and, as a result, the form factor gets suppressed, leading to a smaller decay width.

It thus appears that the expected widths for baryon number violating neutron decay to a dark matter particle inside the nucleus are much larger than that expected in the case of the free neutron [6], provided that its mass is around 938 MeV.

Note that the uncertainty due to the experimental error (1%) is far below those due to the theoretical evaluations. Note the recent experimental limit on $n \rightarrow \gamma\chi$ [30].

It is finally remarked that the experimental width for ^{11}Be is smaller by factors around $10 - 3$ than that evaluated for $n \rightarrow \chi$ with $m_\chi \leqslant 939$ MeV in ^{11}Be, and thus the free neutron decay of $n \rightarrow \chi\gamma$ with $m_\chi = 937.90$ Mev–938.78 of [6] can hardly be the major channel to account for the neutron lifetime discrepancy between the 'bottle' and 'beam' experiments.

Appendix A. Evaluation of the nuclear matrix element

We need the structure of the initial $A(N, Z)$ nucleus and the structure of the $A(N - 1, Z)^*$ final nucleus. The essential information is contained in the CFP

$$\langle A(N-1, Z)J_f; j(n); A(N, Z)\rangle$$

which separates out the interacting neutron indicated by the quantum numbers $n_r\ell j$ and essentially gives the overlap involving the non interacting nucleons or spectroscopic factor. This can be obtained by a nuclear structure calculation or in some cases extracted from experiment in reaction involving a knock out neutron. Then the matrix element involved is:

$$\mathrm{ME} = \kappa_{\text{scale}}\, \langle A(N-1, Z)J_f; j(n); A(N, Z)\rangle \langle J_f M_i - mjm|J_i M_i\rangle J_{me} \tag{27}$$

where J_{me} will be defined in equation (28) below.

A.1. The elementary transition matrix element

Let us suppose that the decaying nucleon is in a shell model state $n_r\ell j\ m\ (\mathbf{r})$, where r is identified with the enter of mass coordinate of three quark system, i.e.

$$\mathbf{r} \equiv \mathbf{R}_{cm} = \frac{1}{3}(\mathbf{r}_1 + r_2 + r_3) = \frac{\mathbf{R}}{\sqrt{3}}$$

and the outgoing dark mater particle is a spin state m_s. We must first evaluate the J_{me}

$$J_{me} = \langle n_r \ell jm|e^{iq\cdot\mathbf{r}}|m_s\rangle = 4\pi \sum_{\ell' m'} (i)^{\ell'} \langle n_r \ell jm|j_{\ell'}(qr) Y^{\ell'}_{m'}(\hat{r}) m_s\rangle (Y^{\ell'}_{m'}(\hat{q}))^* \tag{28}$$

where $j_{\ell'}(z)$ is a spherical Bessel function and $Y^{\ell'}_{m'}(\hat{r})$ the usual spherical harmonic. Using the standard angular momentum re-coupling we obtain a contribution only when $\ell' = \ell$, $m' = m_s - m$. i.e.:

$$J_{me} = 4\pi f_{j,\ell}\, 2\sqrt{2}\, a_N \sqrt{a_N}\, F_{n_r\ell}(\sqrt{2}\, q a_N) \langle j,\, m,\, \ell,\, m_s - m | 1/2 m_s \rangle (-1)^\ell (Y^\ell_{m_s - m}(\hat{q}))^*, \tag{29}$$

where $f_{j,\ell}$ is the needed angular momentum re-coupling factor given by

$$f_{j,\ell} = \begin{cases} 1/\sqrt{2\ell + 2}, & j = \ell + 1/2 \\ 1/\sqrt{2\ell}, & j = \ell - 1/2 \end{cases} \tag{30}$$

and a_N is the nuclear harmonic oscillator size parameter and

$$F_{n_r\ell}(\sqrt{2}\, q a_N) = \int_0^\infty x^2 \mathrm{d}x \psi_{n_r\ell}(x) j_{\ell'}(q\sqrt{2}\, a_N x),\; x = \frac{r}{a_N\sqrt{2}} \tag{31}$$

(dimensionless 'form factor').

$$F_{0s}(\alpha) = \frac{1}{2}\sqrt[4]{\pi}\, \mathrm{e}^{-\frac{\alpha^2}{4}} \tag{32}$$

$$F_{0p}(\alpha) = \frac{\pi^{1/4}}{2\sqrt{6}} \alpha \mathrm{e}^{-\frac{\alpha^2}{4}} \tag{33}$$

$$F_{0d}(\alpha) = \frac{\sqrt[4]{\pi}\, \alpha^2 \mathrm{e}^{-\frac{\alpha^2}{4}}}{2\sqrt{15}} \tag{34}$$

$$F_{1s}(\alpha) = -\frac{\sqrt[4]{\pi}\, \mathrm{e}^{-\frac{\alpha^2}{4}}(\alpha^2 - 3)}{2\sqrt{6}} \tag{35}$$

A.2. The invariant amplitude squared

The next step is to obtain $|\mathrm{ME}|^2$ average over the initial m-sub-sates and sum over the final m-sub-states. The result is

$$\begin{aligned} \langle |\mathrm{ME}|^2 \rangle &= (\kappa_{\text{scale}} \langle A(N-1, Z) J_f; j(n); A(N, Z) \rangle 4\pi f_{j,\ell}\, 2\sqrt{2}\, a_N \sqrt{a_N}\, F_{n_r\ell}(\sqrt{2}\, q a_N))^2 \\ &\times \frac{1}{2J_i + 1} \sum_{M_i, m} \langle J_f M_i - m j m | J_i M_i \rangle^2 \langle j,\, m,\, \ell,\, m_s - m | 1/2 m_s \rangle^2 Y^\ell_{m_s - m}(\hat{q}) (Y^\ell_{m_s - m}(\hat{q}))^* \\ &= (\kappa_{\text{scale}} \langle A(N-1, Z) J_f; j(n); A(N, Z) \rangle 4\pi f_{j,\ell}\, 2\sqrt{2}\, a_N \sqrt{a_N}\, F_{n_r\ell}(\sqrt{2}\, q a_N))^2 \frac{2}{4\pi} \end{aligned} \tag{36}$$

A.3. Neutron decay in the nucleus with photon emission

We will now consider that the neutron before its decay emits a photon via its magnetic moment in a two step process, i.e.

$$J_i \lim_{n \to \gamma n} J_n \lim_{n \to \chi} J_f$$

In this case we have:

$$\Gamma(J_i \to J_f \chi \gamma) = \Gamma_\gamma \frac{1}{\delta E} \Gamma(J_i \to J_f \chi), \tag{37}$$

where Γ_γ is the radiative width, $\Gamma(J_i \to J_f \chi)$ the width for neutron decay inside the nucleus discussed above and δE an energy denominator, essentially the photon energy.

The neutron photon interaction is given by

$$H = \frac{g_s}{2m_n}\sqrt{4\pi\alpha}\,(\mathbf{k} \otimes \sigma). \tag{38}$$

Thus, one finds the corresponding invariant amplitude

$$\mathscr{M}(k)^2 = \left(\frac{g_s}{2}\right)^2 4\pi\alpha \frac{2}{3}\frac{k^2}{m_n^2}\langle||\sigma||\rangle^2, \tag{39}$$

where k is the photon momentum and $\langle||\sigma||\rangle$ is the reduced ME of the spin normalized to $\sqrt{6}$ for $s_{1/2}$-states. We thus find for the width for photon emission:

$$\Gamma_\gamma = \frac{1}{(2\pi)^2}4\pi\int k^2\mathrm{d}k\frac{1}{2k}\mathscr{M}(k)^2\delta(\delta E - k) = \frac{1}{\pi}\left(\frac{g_s}{2}\right)^2 4\pi\alpha\frac{1}{3}\frac{(\delta E)^3}{m_n^2}\langle||\sigma||\rangle^2 \tag{40}$$

(the factor $1/(2k)$ in the first expression comes from the photon normalization).

The total photon width for the neutron decay for photon emission is given by:

$$\Gamma(J_i \to J_f\chi\gamma) = \Gamma(J_i \to J_f\chi)\frac{1}{\pi}\left(\frac{g_s}{2}\right)^2 4\pi\alpha\frac{1}{3}\frac{(\delta E)^2}{m_n^2}\langle||\sigma||\rangle^2. \tag{41}$$

we thus obtain for $1s_{1/2}$ neutron and $\delta E = m_n - m_\chi - B$:

$$\frac{\Gamma(J_i \to J_f\chi\gamma)}{\Gamma(J_i \to J_f\chi)} = \left(\frac{g_s}{2}\right)^2\frac{2}{\pi}4\pi\alpha\frac{(m_n - B - m_\chi)^2}{m_n^2}. \tag{42}$$

Or

$$\frac{\Gamma(J_i \to J_f\chi\gamma)}{\Gamma(J_i \to J_f\chi)} \approx 2.4 \times 10^{-7}\frac{(m_n - B - m_\chi)^2}{1\mathrm{MeV}^2}. \tag{43}$$

Appendix B. Single nucleon wave functions

In this case one performs an isometric transformation [25] of the nucleon wave function $u(r)$:

$$u(r) \to v(r) = s(r)u(f(r)),\ s(r) = \frac{f(r)}{r}\sqrt{\frac{\mathrm{d}f}{\mathrm{d}r}} \tag{44}$$

where $f(r)$ is a continuous function with the properties:

$$f(r) \to r \text{ as } r \to 0,\ f(r) \to \gamma\sqrt{r} \text{ as } r \to \infty,\ \gamma = 2a_N\frac{\sqrt{2m_n\epsilon}}{\hbar} \tag{45}$$

where the parameter ϵ they employed is the binding energy of the neutron considered as positive. i.e. B in our notation. Thus, this transformation achieves

$$\mathrm{e}^{-\frac{r^2}{2a_N^2}} \to \mathrm{e}^{-r\left(\frac{\sqrt{2m_n\epsilon}}{\hbar}\right)}\text{for large } r. \tag{46}$$

Thus, it changes a Gaussian into a Yukawa like behavior. In the case of ^{11}Be a binding energy of 0.8 MeV was previously employed [25]. In the present calculation we will employ the more up to date value of $B = 0.51$ MeV. The authors found it appropriate to employ the following function:

$$f(r, m, \gamma) = \left[\left(\frac{1}{\gamma\sqrt{r}}\right)^m + \left(\frac{1}{r}\right)^m\right]^{(-\frac{1}{m})} \tag{47}$$

with $m = 4$ and $m = 8$. It turns out that for our purposes it does not make much difference which of the two values is adopted.

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References

[1] Wietfeldt F E and Greene G L 2011 *Rev. Mod. Phys.* **83** 1173
[2] Greene G L and Geltenbort P 2016 *Sci. Am.* **314** 36
[3] Patrignani C *et al* (Particle Data Group) 2016 *Chin. Phys.* C **40** 1000001
[4] Byrne J and Dawber P G 1996 *Europhys. Lett.* **33** 187
[5] Yue A T, Dewey M S, Gilliam D M, Greene G L, Laptev A B, Nico J S, Snow W M and Wietfeldt F E 2013 *Phys. Rev. Lett.* **111** 222501
[6] Fornal B and Grinstein B 2018 *Phys. Rev. Lett.* **120** 191801
[7] Motta T F, Guichon P A M and Thomas A W 2018 *J. Phys. G.: Nucl. Part. Phys.* **45** 05LT01
[8] McKeen D, Nelson A E, Reddy S and Zhou D 2018 *Phys. Rev. Lett.* **121** 061802
[9] Baym G, Beckand D H, Geltenbort P and Shelton J 2018 *Phys. Rev. Lett.* **121** 061801
[10] Cline J M and Cornell J M 2018 *J. High Energy Phys.* JHEP07(2018)081
[11] Ejiri H 1993 *Phys. Rev.* C **48** 48
[12] Hazama R, Ejiri H, Fushimi K and Ohsumi H 1994 *Phys. Rev.* C **49** 2407
[13] Czarnecki A, Marciano W J and Sirlin A 2018 *Phys. Rev Lett.* **120** 202002
[14] Schmitt K *et al* 2012 *Phys. Rev. Lett.* **108** 192701
[15] Hammer H-W and Phillips D R 2011 *Nucl. Phys.* A **865** 17
[16] Calci A, Navratil P, Roth R, Dohet-Eraly J and Quaglioni S 2016 *Phys. Rev Lett.* **117** 242501
[17] Baye D, Capel P and Goldstein G 2005 *Phys. Rev. Lett.* **95** 082502
[18] Capel P and Nunes F M 2006 *Phys. Rev* C **73** 014615
[19] Capel P and Nunes F M 2007 *Phys. Rev* C **75** 054609
[20] Otsuka T, Fukunishi N and Sagawa H 1993 *Phys. Rev. Lett.* **70** 1385
[21] Forssén C, Navratil P, Ormand W E and Caurier E 2005 *Phys. Rev.* C **71** 044312
[22] Winfield J S *et al* 2001 *Nucl. Phys.* A **683** 48
[23] So S I and Strottman D 1976 *J. Math. Phys.* **20** 153
[24] Aoki E S Y, Izubuchi T and Soni A 2017 *Phys. Rev.* D **96** 014506
[25] Karataglidis S and Amos K 2005 *Phys. Rev* C **71** 064601
[26] Gross J D, Jolivette P L, Brown C P, Darden S E, Weller H R and Blue R A 1975 *Phys. Rev.* C **12** 1730
[27] Firestone R C (ed) 1999 *Table of Isotopes* 8th edn (New York: Wiley)
[28] Alburger D and Engelfertink G 1970 *Phys. Rev* C **2** 1594
[29] Pfützner M and Riisager K 2018 *Phys. Rev.* C **97** 042501
[30] Tang Z *et al* 2018 *Phys. Rev. Lett.* **121** 022505